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ON FRICTION RESISTANCE TO MOTION OF AN AEROSTATIC CARRIER WITH ELASTIC DIAPHRAGM*

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The motion of an aerostatic carrier smooth elastic diaphragm on a rough supporting surface is considered. A semi-empirical dependence of the friction coefficient on the supporting surface roughness, air flow rate, load, carrier size, and on the diaphragm material elastic properties is obtained on the basis of theoretical analysis of the effect of the diaphragm flexural rigidity on the contact section size, and of experimental investigation results.

Aerostatic carriers (ASC) used for transporting loads /1—5/ have as their basic component an elastic diaphragm that adapts itself to local unevenness of the supporting surface, maintaining a constant gap between the latter and the diaphragm, thus preventing excessive escape of air from the air cushion. But owing to the gap smallness (0.03-0.1mm), the diaphragm touches individual projections of the microcontour of the supporting surface, which results in the generation of dry friction during movement of the diaphragm. The section subjected to friction is of the order of the diaphragm thickness. To calculate the length of that section it is necessary to take into account the diaphragm flexural rigidity. Experiments revealed a complex dependence of the resistance coefficient on load, air flow rate, and ASC parameters.

Note that such investigations are necessary not only in engineering calculations of the traction force but, also, in the investigation of stability of equipment carried on the air cushion over rough surfaces.

1. Asymptotic analysis of viscous flow in a layer with elastic boundary taking into account flexural rigidity. Consider the Kármán equations for large deflections of round plates

$$D\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}\right]\right\}-\frac{1}{r}\frac{d}{dr}\left(rN_r\frac{dw}{dr}\right)=\Delta p \tag{1.1}$$

$$r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r^2N_r)\right]+\frac{E\delta}{2}\left(\frac{dw}{dr}\right)^2=0,\quad D=\frac{E\delta^3}{12}$$

where shear forces are assumed zero, w and δ are, respectively, the vertical deflection and thickness of the diaphragm, E is the Young modulus, D is the flexural rigidity, Δp is the difference of pressures on opposite sides of the diaphragm, N_r is the radial tension, r is the distance from the axis of symmetry z (Fig.1), and a and b are the radii of the inner and outer diaphragm rigid seals, respectively.

The diaphragm cross section drawn through its vertical axis of symmetry is shown in Fig. 1, where p_a denotes atmospheric pressure, p_b and p_c denote pressure in the baloon and in the air cushion, respectively, r_0 is the distance of minimal gap from the axis of symmetry, h_0 is the minimal gap size, and $h(r)$ defines the diaphragm elevation realative to the supporting surface.

Equations (1.1) must be supplemented by the Reynolds equation that defines the flow of a viscous fluid in the minimal gap zone

$$\partial p/\partial r = 6\mu Q/(\pi r h^3) \tag{1.2}$$

where p is the pressure, μ is the dynamic viscosity of gas (for air $\mu = 1.8\cdot 10^{-6}$ kg/m^{2}s), Q is the volume rate of air flow, and h is the vertical distance of a diaphragm point from the supporting surface.

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In the lubrication zone close to the diaphragm maximum deflection tension varies insignificantly, and the viscous layer size is small in comparison with the distance r_0 radius of the diaphragm maximum deflection from the z axis. Hence. as in /4/, we substitute in Eqs.(1.1) and (1.2) N_0 for N_r and r_0 for r, where N_0 is the tension at the maximum deflection point.

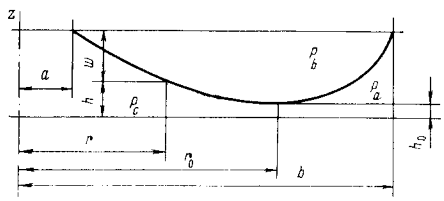

Fig.1

Since $h + w =$ const, we can eliminate pressure from Eqs.(1.1) and (1.2).

Passing to dimensionless variables $x = (r - r_0)/\lambda$, $y = h/h_0$, where h_0 is the minimum gap between the diaphragm and the supporting surface at the point of maximum diaphragm deflection $\lambda^3 \equiv \pi r_0 N_0 h_0^4/(6\mu Q)$, we obtain for the deflection an equation of the form

$$-\alpha^2 \frac{d^5y}{dx^5} + \frac{d^3y}{dx^3} = \frac{1}{y^3}, \quad \alpha^2 \equiv \frac{E\delta^3}{12N_0\lambda^2} \tag{1.3}$$

Parameter λ defines the length of the section of viscous force action, in which there is a sharp pressure drop /4,5/. Besides this characteristic length there is the characteristic length $l = \delta\sqrt{E\delta/N_0}$ of the section subjected to bending moments. The coefficient α in Eq. (1.3) is the ratio $\alpha = l/\lambda$ of these two lengths.

The variable x plays the part of internal variable in the viscous layer. In the internal expansion, where the relative variation of tension $\Delta N_r/N_0$ and of radius $\Delta r_0/r_0$ inside the viscous layer are small parameters, part of the boundary conditions is defined asymptotically with $x \to +\infty$ and $x \to -\infty$. As the boundary conditions we take that pressure in the cushion approaches p_c as $x \to -\infty$, and as $x \to +\infty$ it approaches the atmospheric pressure p_a /4/.

In Eq.(1.1) $\Delta p = -(p_b - p)$, where p_b is the pressure in the balloon (above the diaphragm) and p is the current pressure in the cushion (under the diaphragm). On the basis of the first of Eqs.(1.1) we have

$$\lim_{x\to-\infty}\left(-\alpha^2 \frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}\right) = \theta_2, \quad \theta_2 \equiv \frac{p_b - p_c}{N_0h_0}\lambda^2 \tag{1.4}$$

$$\lim_{x\to+\infty}\left(-\alpha^2 \frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}\right) = \theta_1, \quad \theta_1 \equiv \frac{p_b - p_a}{N_0h_0}\lambda^2$$

At $x = 0$ the following conditions apply:

$$y(0) = 1, \; y'(0) = 0 \tag{1.5}$$

When $|r - r_0| \gg l$ the effect of bending moments is exponentially small. Hence for $x \to \pm\infty$ we use the additional condition

$$x \to \pm\infty, \qquad d^4y/dx^4 \to 0 \tag{1.6}$$

implying that the exponentially increasing solutions must vanish.

Since Eq.(1.3) is of the fifth order and the number of conditions in (1.4), (1.5), and (1.6) is six, hence for any arbitrary values of the right-hand sides of θ_1 and θ_2 in (1.4) these conditions are incompatible, except when θ_1 and θ_2 have well-defined values that depend only on the ration $(p_b - p_c)/(p_b - p_a) = q$ and coefficient α, which enables us to establish the dependence of the minimal gap on the parameters of ASC.

Integrating Eq.(1.3) and using condition (1.4) for $x \to +\infty$, we obtain

$$-\alpha^2 y^{(4)} + y'' = \theta_1 - \int_x^\infty \frac{dt}{y^3} \tag{1.7}$$

Similarly, from condition (1.4) with $x \to -\infty$ follows

$$-\alpha^2 y^{(4)} + y'' = \theta_2 + \int_{-\infty}^x \frac{dt}{y^3} \tag{1.8}$$

Compatibility of Eqs.(1.7) and (1.8) requires the fulfillment of the equality

$$\int_{-\infty}^\infty \frac{dt}{y^3} = \theta_1 - \theta_2 \tag{1.9}$$

which is the missing condition needed for the determination of constants θ_1 and θ_2, since in conformity with (1.4) their ratio is assumed known.

Rejecting in (1.7) and (1.8) the exponentially increasing solutions as $x = \pm\infty$, we obtain

$$y'' = \frac{1}{2}\left\{(\theta_1 + \theta_2) - \int_x^\infty \frac{dt}{y^3}\left(1 - e^{(x-t)/\alpha}\right) + \int_{-\infty}^x \frac{dt}{y^3}\left(1 - e^{(t-x)/\alpha}\right)\right\} \tag{1.10}$$

Differentiating (1.10) we obtain for y'''

$$y''' = \frac{1}{2\alpha}\left\{\int_x^\infty \frac{dt}{y^3(t)} e^{(x-t)/\alpha} + \int_{-\infty}^x \frac{dt}{y^3(t)} e^{(t-x)/\alpha}\right\} \tag{1.11}$$

Let us consider the cases of $\alpha \gg 1$ and $\alpha \ll 1$ separately.

When $\alpha \ll 1$ for the determination of integrals in the right-hand side of (1.11) we apply Watson's lemma /6/, according to which the equality

$$\int_0^\infty e^{-t/\alpha} f(t)\, dt \approx \sum_{n=0}^\infty f^{(n)}(0)\, \alpha^{n+1} (-1)^n$$

is valid and accurate to exponentially small terms. Its right-hand side is a Maclaurin expansion of function $f(t)$ within its radius of convergence. Hence from (1.11) within terms of order $O(\alpha^4)$ we have

$$y''' = y^{-3} + \alpha^2 d^2 y^{-3}/dx^2 \tag{1.12}$$

The dependence $Y = (p_0 - p)\lambda^2/(N_0 h_0)$ obtained with flexural rigidity taken into consideration is shown in Fig.2 by the dash line, and the solid line corresponds to calculations without allowance for flexural rigidity. The use of bending stresses as the small parameter in the hydroelastic problem shows that in the external part of the viscous layer the theoretically determined pressure is lower than the atmospheric.

This fact, established experimentally, was earlier explained only by the effect of forces of the stream inertia /1/. The characteristic length of the section in which friction takes place is of the order of λ. On the basis of numerical calculations for $l \ll \lambda$ it is possible to assume that the characteristic length of contact section is equal $l + \lambda$, with θ_1 varying in the interval 1.8 − 2 for small q.

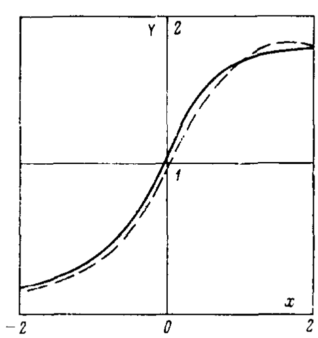

Fig.2

Consider now the case of $\alpha \gg 1$ when the characteristic length λ of pressure drop section is considerably small than length l. It is possible to assume in the first approximation the pressure to be piecewise constant. Then from (1.7) and (1.8) follows that

$$-\alpha^2 y^{(4)} + y'' = \begin{cases} \theta_1 & \text{when } x > 0 \\ \theta_2 & \text{when } x < 0 \end{cases} \tag{1.13}$$

The condition of continuity of y'' and y''' implies that at $x = 0$ we have

$$\begin{aligned} x > 0,\ y'' &= \theta_1 - (\theta_1 - \theta_2)\, e^{-x/\alpha}/2 \\ x < 0,\ y'' &= \theta_2 + (\theta_1 - \theta_2)\, e^{x/\alpha}/2 \end{aligned} \tag{1.14}$$

from which with condition (1.6) and $x = 0$ follows that

$$x > 0,\ y = 1 + {}^1/_2\theta_1 x^2 - {}^1/_2(\theta_1 - \theta_2)(\alpha x - \alpha^2 + \alpha^2 e^{-x/\alpha}) \tag{1.15}$$

$$x < 0,\ y = 1 + {}^1/_2\theta_2 x^2 + {}^1/_2(\theta_1 - \theta_2)(-\alpha x - \alpha^2 + \alpha^2 e^{x/\alpha}) \tag{1.16}$$

It remains to determine θ_1 and θ_2 using (1.9). The integral in it is calculated asymptotically. For this we divide the integration region $(-\infty, +\infty)$ in three parts, viz. $(-\infty, -\alpha)$, $(-\alpha, \alpha)$, and (α, ∞). Experimental data enable us to assume that $q \equiv \theta_2/\theta_1 \ll 1$.

When integrating from $-\infty$ to α in the interval (1.16), we reject the exponent and set $\theta_2 = 0$. In region $(-\alpha, \alpha)$ we expand exponents in formulas (1.15) and (1.16) and take into account the first three terms. The integral from α to ∞ is of order α^{-5} and is neglected. For the determination of θ_1 we finally obtain the equation

$$^{3}/_{4}\pi\theta_1^{-1/2}(1 - q/2) + (\theta_1\alpha)^{-1} = \theta_1 (1 - q)$$

from which follows within terms of order q^2 and $1/\alpha^2$ that

$$\theta_1 = \theta_1 (\alpha, q) = \left(\frac{3}{4}\pi + \frac{2}{\alpha\sqrt{3\pi}} + \frac{3}{4}\pi q\right)^{2/3} \tag{1.17}$$

Calculations by formula (1.17) differ insignificantly from numerical solutions of Eq. (1.9) in which expressions (1.15) and (1.16) have been substituted for y.

Using formula (1.4) for θ_1 for the minimal gap h_0 we obtain

$$h_0 = \left(\frac{6\mu Q}{\pi r_0 N_0}\right)^{2/5} \left(\frac{\theta_1 N_0}{p_b - p_a}\right)^{3/5} \tag{1.18}$$

Thus the allowance for flexural rigidity affects only slightly the minimal gap magnitude. But the characteristic length of the section along which dry friction of the diaphragm on the supporting surface develops, is determined by the square root of the ratio of flexural rigidity to tension at the minimal gap point under the condition that $l \gtrsim \lambda$.

2. Determination of the friction coefficient dependence on parameters of ASC in motion. We denote by χ the coefficient of friction per unit area of contacting surfaces. This coefficient depends only on the ratio of the local gap h to the characteristic dimension of a projection of the floor rough surface. The /traction/ resistance force per unit area is thus equal $(p_b - p)\chi$. Integrating this expression over the whole surface of the floor and diaphragm contact, taking into account that $l \ll r_0$, we obtain for the traction force the expression

$$2\pi r_0 \int\limits_{r_0 - l/2}^{r_0 + l/2} (p_b - p)\, \chi\, dr$$

The coefficient of friction k for the carrier as a whole is equal to the ratio of the tractive effort to the aerostatic carrier weight G (including the paylod)

$$k = 2\pi r_0 G^{-1} \int (p_b - p)\chi dr \sim 2\pi r_0 G^{-1} (p_b - p_a)\chi l = 2l\chi_0/r_0 \tag{2.1}$$

where χ_0 is the value of χ when $h = h_0$ (h_0 is the minimal gap).

For the determination of the friction coefficient in (2.1) it is necessary to determine the dependence of χ_0 on the roughness of the supporting surface. Since theoretical determination of that dependence is difficult, it was evaluated on the basis of experimental data. Investigations were carried out on equipment consisting of three and four aerostatic carriers. Dimension b, i.e. the radius of the diaphragm external seal, was from 0.18 to 0.4 m. The over-all load on the equipment was from 0.7 to 30 ton, and the air volume flow rate was varied from 0.3 to 6 m^3/min. The supporting surface roughness measured by special equipment was $R_z = 0.02-0.08$ mm.

The dependence of χ_0 on h_0/R_z is shown in Fig.3, where the small circles, triangles, and squares represent the values of χ_0 determined experimentally for ASC of the following sizes $b = 0.182, 0.308$, 0.46 m, respectively.

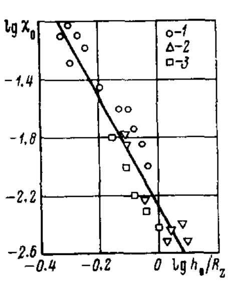

Fig.3

Note the grouping of experimental points along the straight line defined by the equation

$$\lg \chi_0 = a - \varkappa \lg (h_0/R_z); \quad a = -2, \quad \varkappa = 3.5 \tag{2.2}$$

where R_z is the floor roughness.

Parameter χ_0 determined by formula (2.2) based on experimental data depends only on the ratio h_0/R_z, and is independent of external load, air flow rate, external seal radius, and of the diaphragm thickness and material.

Using expression (2.1) for the friction coefficient, the empirical formula (2.2), the previously adduced formula $l = \delta\sqrt{E\delta/N_0}$, and the expression $N_0 = n^3\sqrt{G^2E\delta/b^2}$, where $0.11 < n < 0.15$, from /1/, for the determination of the friction coefficient we obtain

$$k = s\frac{\delta}{r_0}\left(\frac{E\delta b}{G}\right)^{1/3}\left(\frac{R_z}{h_0}\right)^{\varkappa}$$

For the determination of the ratio h_0/R_z we use formula (1.18) written in the form

$$\frac{h_0}{R_z} = t\,\frac{r_0}{R_z}\left(\frac{\mu E\delta Q}{G^2}\right)^{2/5}\left(\frac{G}{E\delta b}\right)^{1/3}$$

The coefficients s and t are within the limits $0.11 < s < 0.13,\ 2.7 < t < 3$, respectively. For the determination of r_0 the data in /5/ are to be used.

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